CSCE 689: Special Topics in Modern Algorithms for Data Science

Lecture 34

Samson Zhou

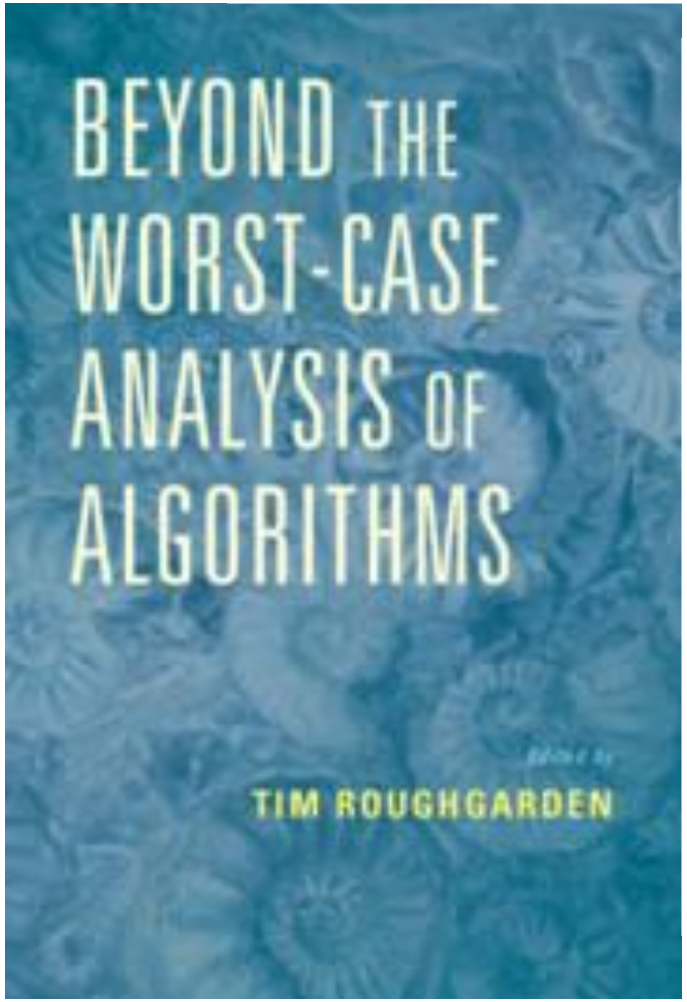
Presentation Schedule

- November 27: Chunkai, Jung, Galaxy AI
- November 29: STMI, Anmol, Jason
- December 1: Bokun, Ayesha, Dawei, Lipai

Stellar™ 6.890 Learning-Augmented Algorithms

LOGIN

Course : » Course 6 : » Spring 2019 : » 6.890 : »Materials



Learning-Augmented Algorithms

- For a certain task and input, algorithm is given advice
- Advice could be “good”, advice could be “bad”
- Goal: “Good” performance if the advice is good, “normal” performance if the advice is bad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Mahabalipuram
73°
Partly Cloudy
H:86° L:72°
Sunny conditions expected around 9AM.
Now 8AM 9AM 10AM 11AM 12P
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10-DAY FORECAST
Today 72° 86°
Fri 72° 86°
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10-DAY FORECAST
Today 73° 85°
Fri 73° 85°
Sat 72° 84°
Sun 72° 84°

Learning-Augmented Algorithms

- Better data structures: Bloom filters with lower false positive rates [Mitzenmacher18]
- Better space-accuracy tradeoff for streaming algorithms: Frequency estimation, e.g., CountMin, CountSketch [HsuIndykKatabiVakilian19], moment estimation, distinct elements [JiangLinRuanWoodruff20], triangle counting [ChenEdenIndykLinNarayananRubinfeldSilwalWagnerWoodruffZhang22]
- Better size-accuracy tradeoff for sketching: Low-rank approximation [IndykVakilianYuan19]

Learning-Augmented Algorithms

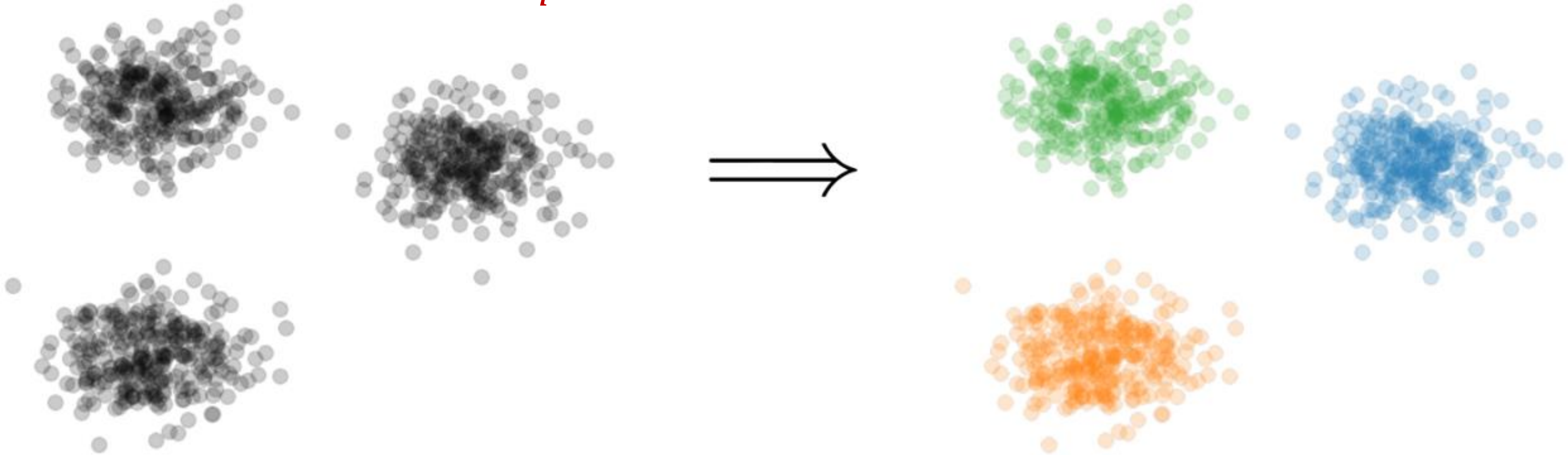
- Warm-start to search algorithms: Binary search [LinLuoWoodruff22], Max-flow [ChenSilwalVakilianZhang22], [DaviesMoseleyVassilvitskiiWang23], matchings [DinitzImLavastidaMoseleyVassilvitskii21]
- Better accuracy-sample complexity tradeoff: Support size estimation [EdenIndykNarayananRubinfeldSilwalWagner21]
- Better online algorithms: Set cover [BamasMaggioriSvensson20], [GrigorescuLinSilwalSongZhou23], Scheduling [LattanziLavastidaMoseleyVassilvitskii20], [ScullyGrosofMitzenmacher22]
- Better privacy-utility tradeoffs for DP: Quantile estimation [KhodakAminDickVassilvitskii23]
- Beating NP-hardness?

Graph Searching with Predictions	Banerjee, Cohen-Addad, Gupta, Li	arXiv '22	exploration, online, search
Scheduling with Predictions	Cho, Henderson, Shmoys	arXiv '22	online, scheduling
On the Power of Learning-Augmented BSTs	Chen, Chen	arXiv '22	data structure, search
Algorithms with Prediction Portfolios	Dinitz, Im, Lavastida, Moseley, Vassilvitskii	arXiv '22	load balancing, matching, multiple predictions, online, scheduling
Private Algorithms with Private Predictions	Amin, Dick, Khodak, Vassilvitskii	arXiv '22	differential privacy
Paging with Succinct Predictions	Antoniadis, Boyar, Eliáš, Favrholdt, Hoeksma, Larsen, Polak, Simon	arXiv '22	caching/paging, online
Proportionally Fair Online Allocation of Public Goods with Predictions	Banerjee, Gkatzelis, Hossain, Jin, Micha, Shah	arXiv '22	allocation, online
Canadian Traveller Problem with Predictions	Bampis, Escoffier, Xefteris	arXiv '22, WAOA '22	online, routing
Learning-Augmented Algorithms for Online Linear and Semidefinite Programming	Grigorescu, Lin, Silwal, Song, Zhou	arXiv '22	covering problems, online, SDP

Learning-Augmented Clustering

- Goal: Given dataset P in d dimensions, output a set C of k centers to minimize

$$\sum_{p \in P} \min_{c \in C} \|p - c\|_2^2$$



Learning-Augmented Clustering

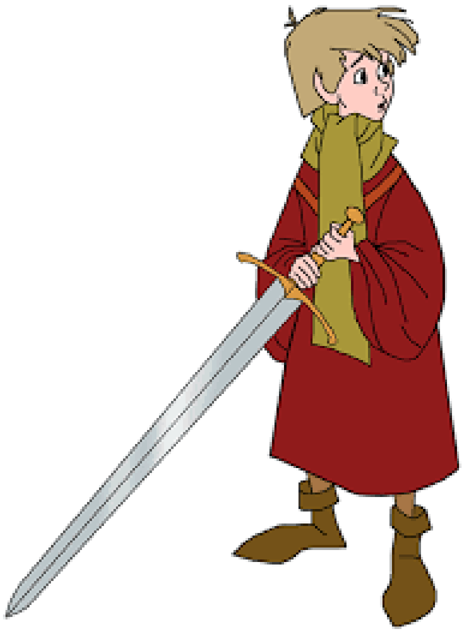
- Goal: Given dataset P in d dimensions, output a set C of k centers to minimize

$$\sum_{p \in P} \min_{c \in C} \|p - c\|_2^2$$

- NP-hard to even approximate within a factor of 1.07 [Cohen-AddadC.S.20, LeeSchmidtWright17]
- Beyond worst-case: Clustering on inputs from some “nice” distribution, similar inputs or inputs with auxiliary information
- Hope: ML can guide the clustering, so we can overcome worst-case with advice!

Predictor

- Suppose Π outputs noisy labels according to a $(1+\alpha)$ approximate clustering C and error rate $\lambda \leq \alpha$



What is the label of x_1?

x_1 belongs to cluster 3

What is the label of x_2?

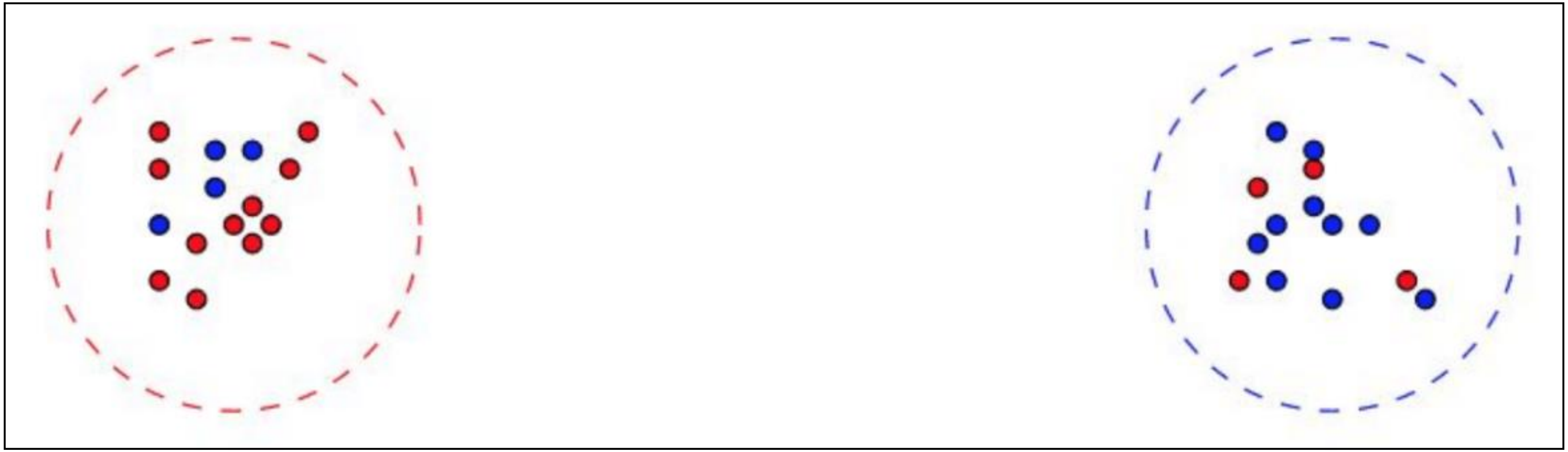
x_2 belongs to cluster 7

Theoretical Guarantee

- Suppose Π outputs noisy labels according to a $(1+\alpha)$ approximate clustering C and error rate $\lambda \leq \alpha$
- Main result [EFSWZ22]: Algorithm that outputs a $(1+O(\alpha))$ approximate k-means clustering in nearly linear time
- "Predictions can overcome complexity hardness barriers!"

Naïve Approach Does Not Work

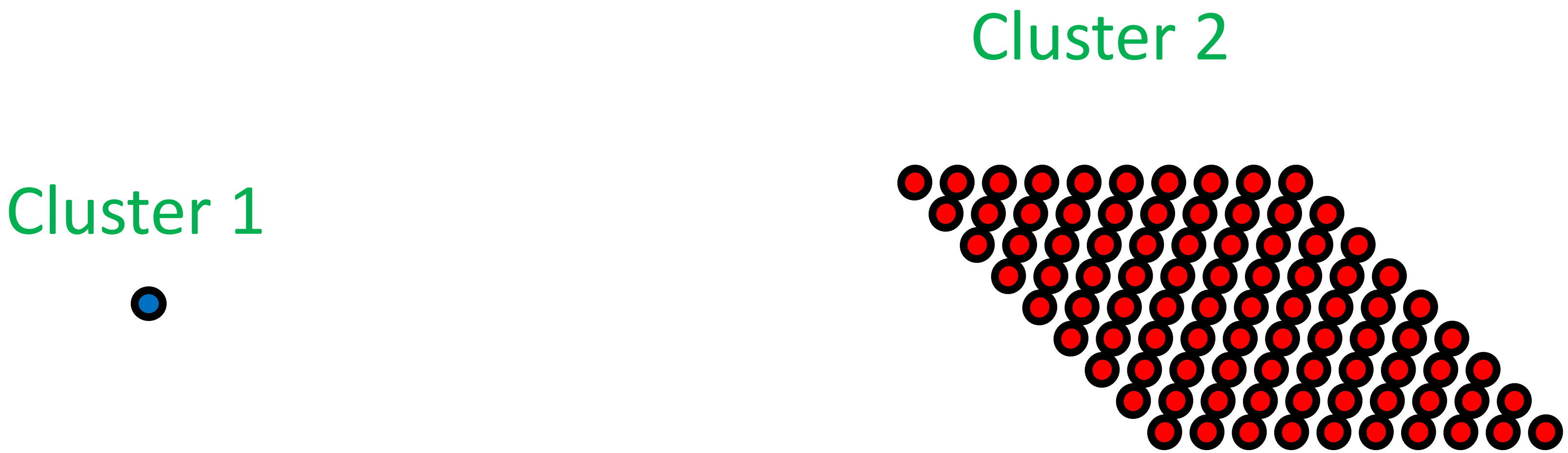
- Not enough to blindly follow predictions!



- Optimal cost ≈ 0
- Predictor with arbitrary small error has large cost!

Naïve Approach Does Not Work

- Can a predictor even help?



Naïve Approach Does Not Work

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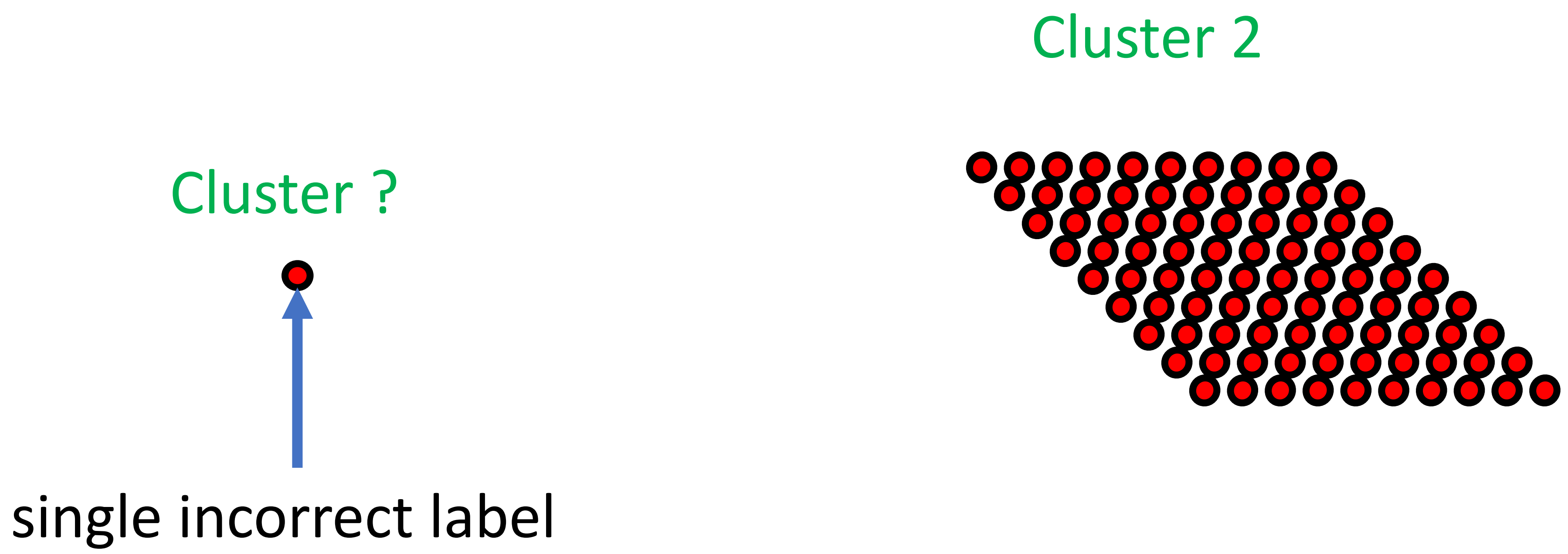
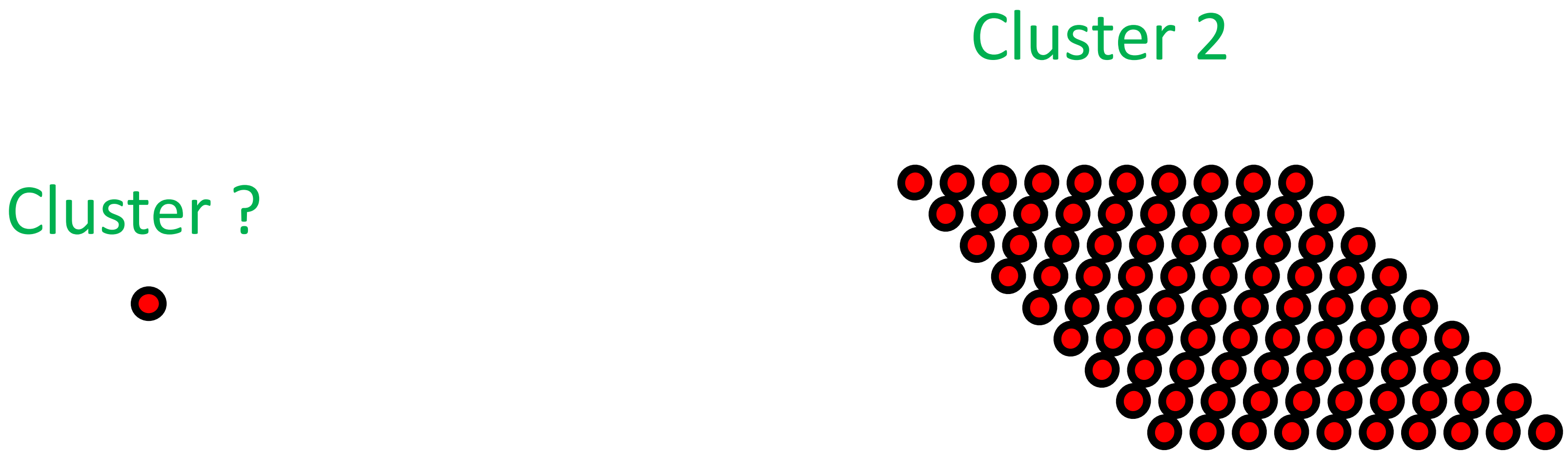


Naïve Approach Does Not Work

- Can a predictor even help?



- MUST have assumptions about the accuracy on each cluster

Precision and Recall

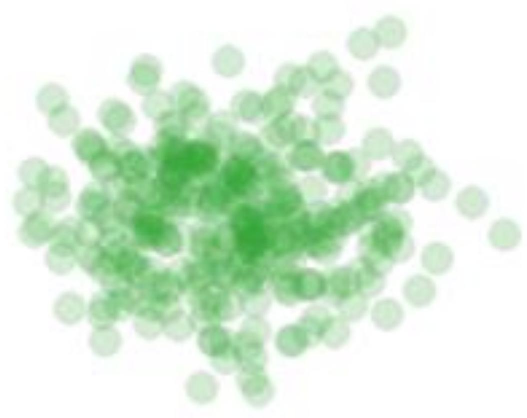
- [EFSWZ22]: Assume cluster sizes are "balanced"

- [NCN23]: Let P_i be the optimal cluster with label i and Q_i be the points that are labeled i. Then $|Q_i \setminus P_i| + |P_i \setminus Q_i| \leq \alpha \cdot |P_i|$.

Precision Recall

Algorithmic Intuition

- Our approach: Closed-form solution for best center of a *fixed* set of points



$$\text{argmin}_c \left[\text{cost}(c, P)\right] = \frac{1}{|P|}\sum_{p\in P} p$$

$$\text{argmin}_c \sum_{p\in P} \|p - c\|_2^2 = \frac{1}{|P|}\sum_{p\in P} p$$

Algorithmic Intuition

- Consider each *dimension* separately

Algorithm 1 Learning-augmented k-means clustering

Input: A point set X with labels given by a predictor Π with label error rate λ

Output: $(1+O(\alpha))$-approximate k-means clustering of X

1: **for** $i = 1$ to $i = k$ **do**

2: Let Y_i be the set of points with label i.

3: Run CRDEST for each of the d coordinates of Y_i.

4: Let C_i' be the coordinate-wise outputs of CRDEST.

5: **end for**

6: **Return** $C_1', \ldots, C_k'$.

Algorithmic Intuition

- Consider each *label* separately

Algorithm 1 Learning-augmented k-means clustering

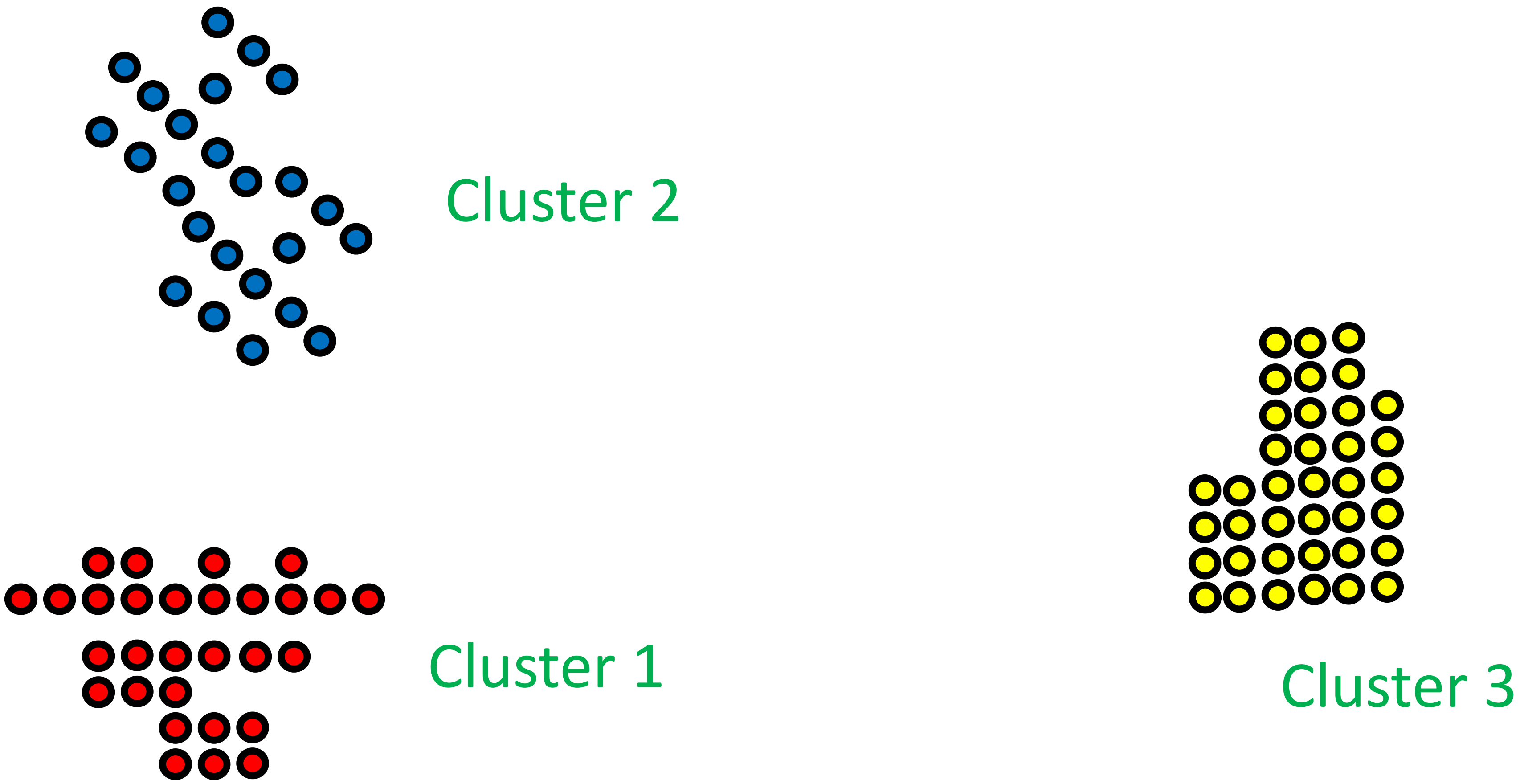
Input: A point set X with labels given by a predictor Π with label error rate λ

Output: $(1+O(\alpha))$-approximate k-means clustering of X

```
for i = 1 to i = k do
    Let Y_i be the set of points with label i.
    Run CrdEst for each of the d coordinates of Y_i.
    Let C'_i be the coordinate-wise outputs of CrdEst.
end for
Return C'_1, ..., C'_k.
```

Algorithmic Intuition

- Example:



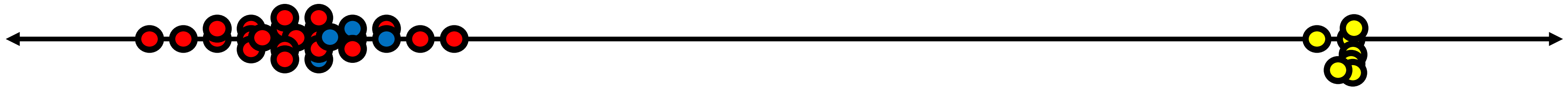
Algorithmic Intuition

- Example: Consider the points with label 1

Algorithmic Intuition

- Example: Consider the points with label 1

- Consider each dimension separately



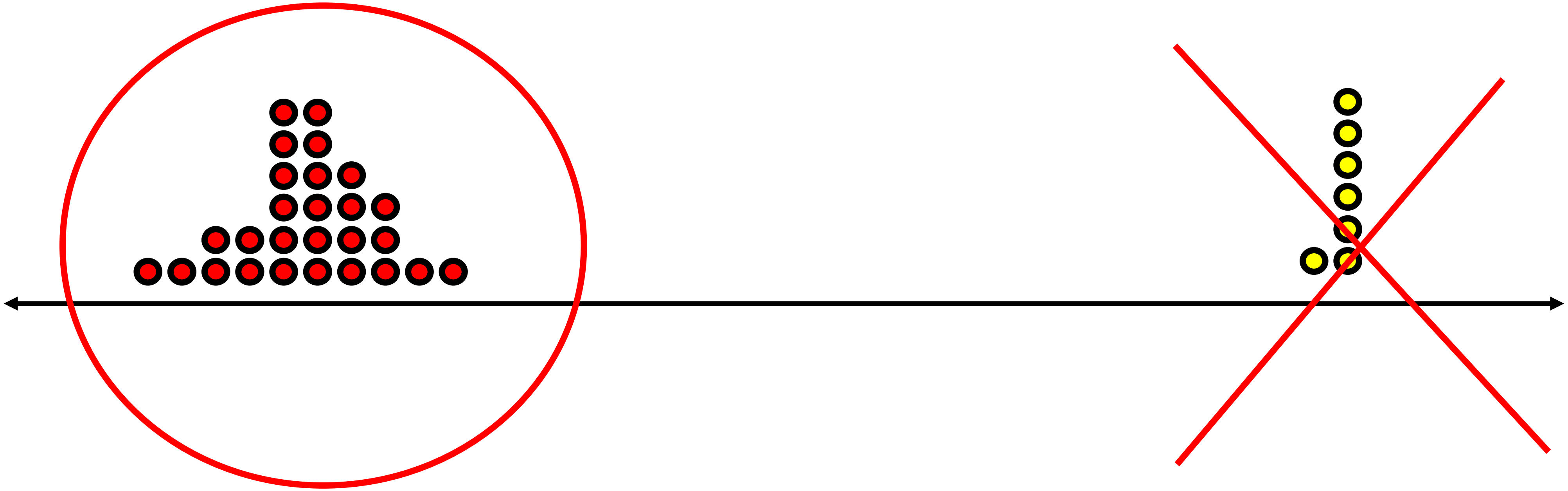
Algorithmic Intuition

- Example: Consider the histogram of points with label 1

Algorithmic Intuition

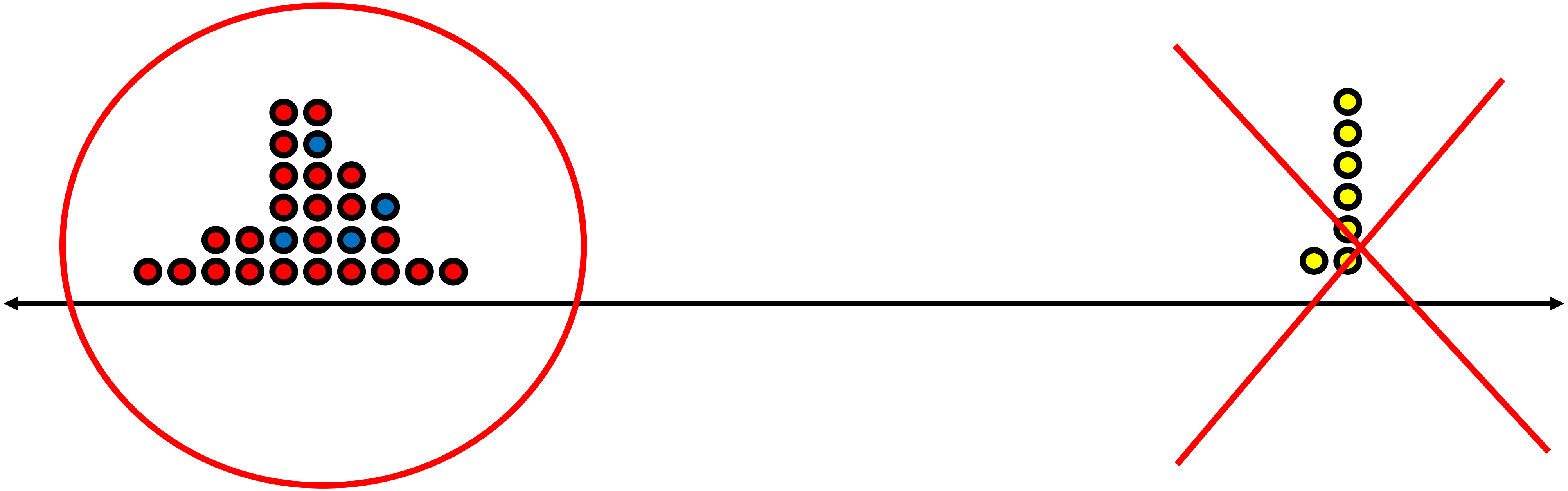
- Example: Consider the histogram of points with label 1

- Is it true that “pruning” away the outliers removes all incorrect points?



Algorithmic Intuition

- Is it true that “pruning” away the outliers removes all incorrect points? NO!



Algorithmic Intuition

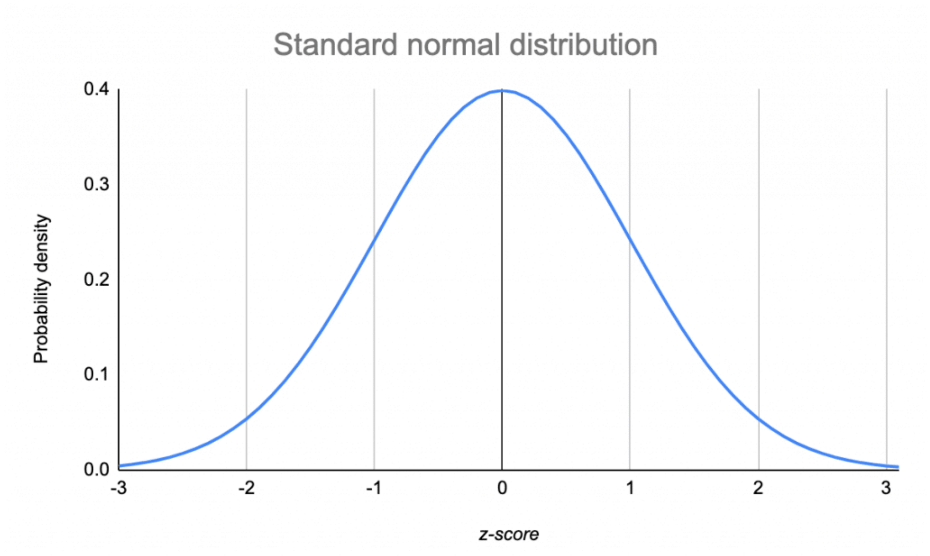
- Example: Consider the points with label 1

Algorithmic Intuition

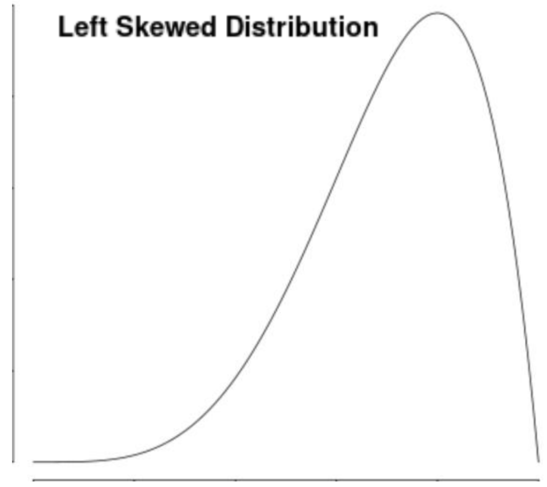
- Consider each label and each dimension separately

- Our approach: Use ideas from robust mean estimation

$(1-\alpha)P$



αQ



Algorithmic Intuition

- Case 1: Q is “far” from P

Algorithmic Intuition

- Case 1: Q is "far" from P

- Can detect handle this case by "pruning" the distribution

Algorithmic Intuition

- Case 2: Q is “close” to P

Algorithmic Intuition

- Case 2: Q is "close" to P

- Q cannot heavily affect the empirical mean P

Algorithmic Intuition

- Algorithm: Find the mean of the shortest interval that contains $(1 - O(\alpha))$ fraction of the points

Algorithm

- Algorithm: Find the mean of the shortest interval that contains $(1 - O(\alpha))$ fraction of the points

Algorithm 2 Coordinate-wise estimation CRDEST

Input: Points $x_1, \ldots, x_{2m} \in \mathbb{R}$, corruption level $\lambda \leq \alpha$

1: Randomly partition the points into two groups X_1, X_2 of size m.

2: Let $I = [a, b]$ be the shortest interval containing $m(1 - 5\alpha)$ points of X_1.

3: $Z \leftarrow X_2 \cap I$

4: $z \leftarrow \frac{1}{|Z|} \sum_{x \in Z} x$

5: **Return** z

Analysis Overview

- Robust mean estimation gives additive α error to the *location* of the mean

- How does this affect the k-means clustering cost?

Analysis Overview

- Analysis: Robust mean gives $(1+\alpha)$-approximation to the 1-means clustering cost
- Recall: Consider each label and each dimension separately

Analysis Overview

- Analysis: Robust mean gives $(1+\alpha)$-approximation to the k-means clustering cost

- Lemma: Let P, Q be sets of real numbers with $|P| \geq (1-\alpha)n$ and $|Q| \leq \alpha n$. Let $X = P \cup Q$, let C_X and C_P be the means of X and P. Then

$$\mathrm{Cost}(X, C_P) \leq (1+\alpha)\mathrm{Cost}(X, C_X)$$

- [InabaKatohImai94]:

$$\mathrm{Cost}(X, C_P) \leq \mathrm{Cost}(X, C_X) + |X| \cdot |C_P - C_X|^2$$

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  Let C'_i be the coordinate-wise outputs of CrdEst.

end for

Return $C'_1, \ldots, C'_k$.

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Randomly partition the points into two groups X_1, X_2 of size m.

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Return z

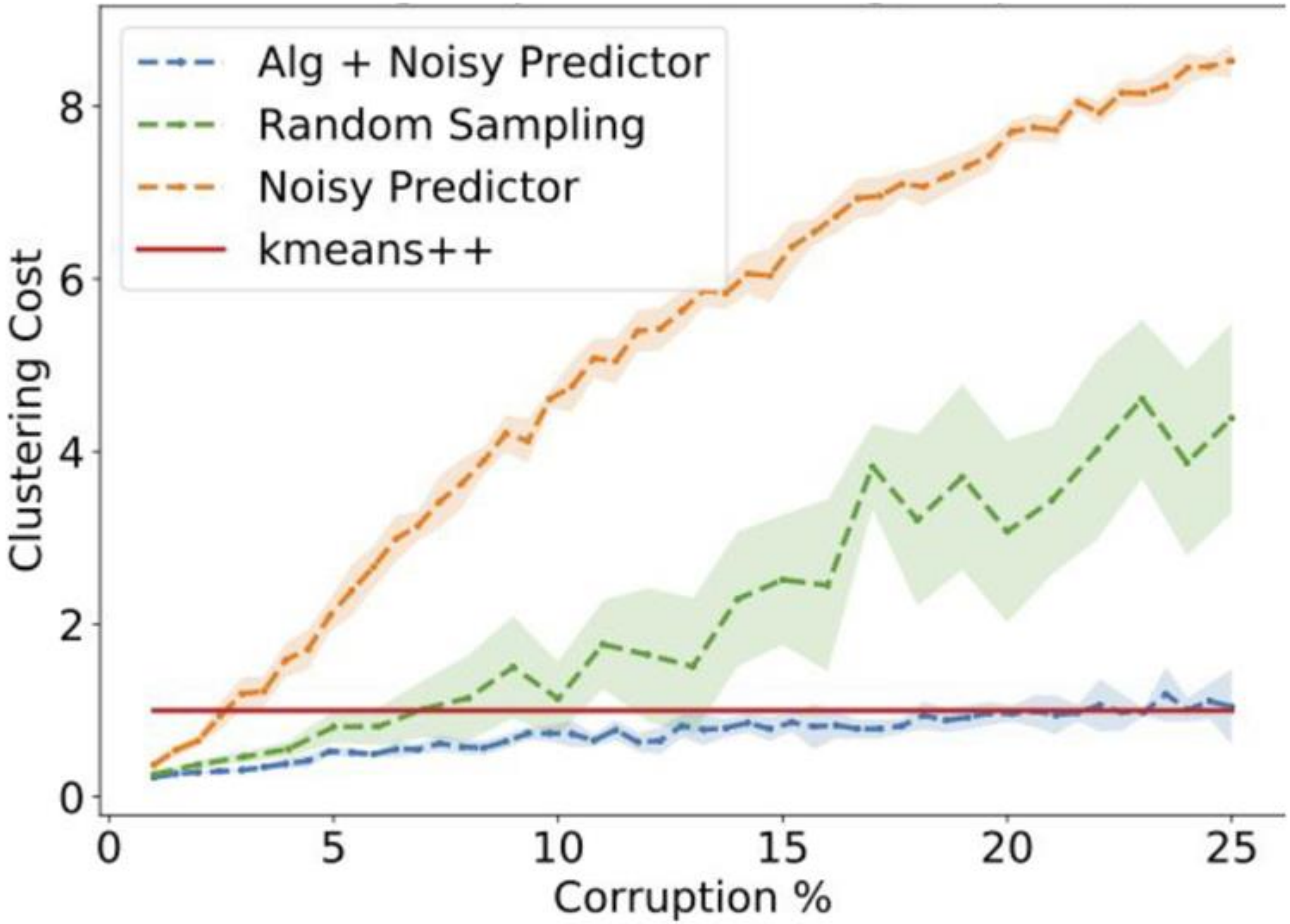
Experimental Results

- Case Study: Spectral clustering on graphs varying over time

- Dataset: Internet router graph varying over the course of a year

- Methodology: Compare to standard benchmarks while using various natural predictors, i.e., noisily perturb true labels and compare to baselines as function of error

Dataset: Internet router graph varying over the course of a year, $k = 10$



Conclusion: Our algorithm (using predictor) outperforms benchmarks such as k-means ++ for low error while staying competitive with high corruptions

Summary

- NP-hard to even approximate within a factor of 1.07 [Cohen-AddadC.S.20, LeeSchmidtWright17]
- Main result [EFSWZ22]: Algorithm that outputs a $(1 + O(\alpha))$ approximate k-means clustering in nearly linear time
- Handles clustering with *outliers*
- Not enough to blindly follow predictions!
- Our approach: Use ideas from robust mean estimation

…and Beyond!

- Related work:
- Semi-supervised active clustering (SSAC) framework: Same cluster queries, [AKB16], [KG17], [MS17], [GHS18], [ABJK18], …, correlation clustering
- Future directions:
- Other predictors (multiple labels per point), relationship with robust statistics, minimizing the number of queries
- Algorithms for (k, z)-clustering, i.e., $\sum_{p \in P} \min_{c \in C} \|p - c\|_2^z$
- Algorithms for L_p-metrics, i.e., $\sum_{p \in P} \min_{c \in C} \|p - c\|_p^p$