

On the Security of Proofs of Sequential Work in a Post-Quantum World

Jeremiah Blocki¹, Seunghoon Lee¹, Samson Zhou²

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²School of Computer Science, Carnegie Mellon University

July 28, 2021

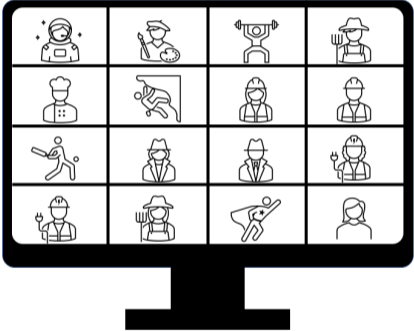
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Mellon
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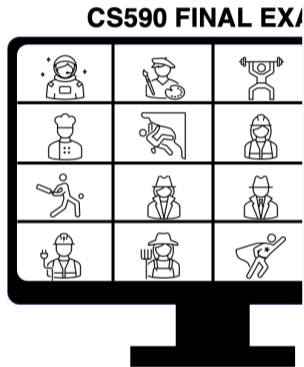
Conference on Information-Theoretic Cryptography (ITC) 2021

Motivation: Online Exams during the Pandemic

CS590 FINAL EXAM



Motivation: Online Exams during the Pandemic



[CS590] 5 mins late - having internet issue

Cinseer Goodman
Tue 5/2/2021 9:05 PM
To: Seunghoon Lee

answer-goodman.pdf
157 KB

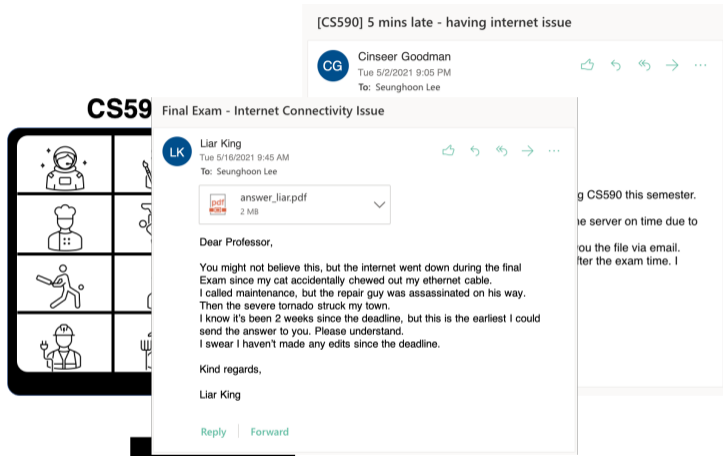
Dear Professor,

My name is Cinseer Goodman who is taking CS590 this semester. I hope this email finds you well. I was not able to submit the final exam to the server on time due to an unexpected internet connectivity loss. It just went back 5 minutes later so I send you the file via email. I promise I have not done any extra work after the exam time. I hope it works. Thank you.

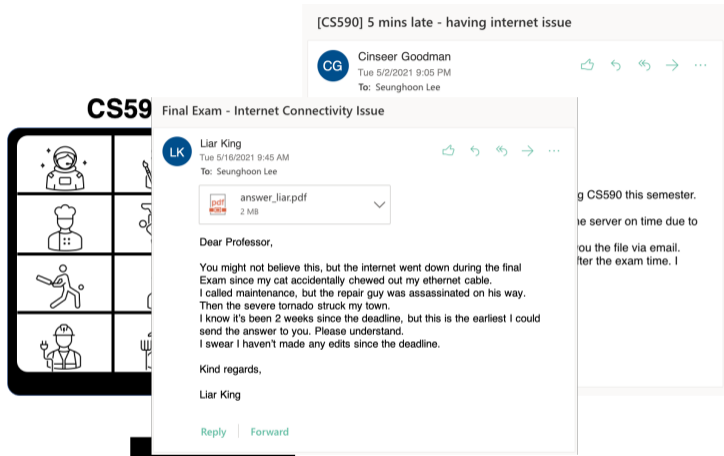
Best,
Cinseer Goodman

[Reply](#) | [Forward](#)

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CS59



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LK Liar King
Tue 5/16/2021 9:45 AM
To: Seunghoon Lee

answer_liar.pdf
2 MB

Dear Professor,

You might not believe this, but the internet went down during the final Exam since my cat accidentally chewed out my ethernet cable. I called maintenance, but the repair guy was assassinated on his way. Then the severe tornado struck my town. I know it's been 2 weeks since the deadline, but this is the earliest I could send the answer to you. Please understand. I swear I haven't made any edits since the deadline.


Kind regards,
Liar King

Reply | Forward

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A **proof** that a large amount () of sequential work was performed after a prover committed an initial message, e.g., the solution for the final exam

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Many late students? insufficient computational resources to verify all solutions

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Soundness: students (prover) should *not* be able to produce a *valid* proof faster (than sequential time , even if running in parallel).

PoSW Constructions

Mahmoody et al. [MMV13]: the first theoretical construction of a PoSW

Verifier time $\tilde{O}(n)$, and prover time $\tilde{O}(n)$,

Parallel cheating prover running in sequential time $\tilde{O}(n)$ cannot fool the verifier, and

Security proof in the **classical ROM**.

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Cohen and Pietrzak [CP18]: an improved & practical PoSW construction

Modular security proof in the **classical ROM**:

Any parallel cheating prover (for the PoSW) must produce a long ϵ -sequence (whp), and

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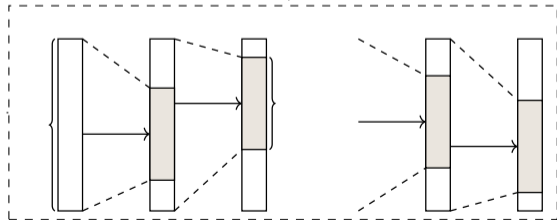
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x_1, x_2, \dots, x_N , s.t.
for each $i \in [N]$, there
exist a_i, b_i such that
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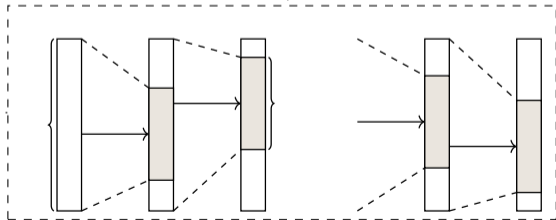
Post-Quantum Security?

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Key Research Questions:

Can a sequentially time-bounded parallel *quantum* attacker produce a long ϵ -sequence?

Can a sequentially time-bounded parallel *quantum* attacker produce a valid PoSW?

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Short answer: NO!

Our Result. Hardness of Producing an ϵ -Sequence/PoSW in a Quantum Setting

Theorem (informal)

A *quantum adversary* making at most q queries over r rounds outputs an ϵ -sequence of length n (with $\epsilon > 0$ where $n > 0$) with *negligible probability* ϵ^{-c} .

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Suppose \mathcal{A} makes at most q quantum queries to the random oracle over at most r rounds. Then \mathcal{A} outputs a valid non-interactive PoSW with *negligible probability* ϵ^{-c} .

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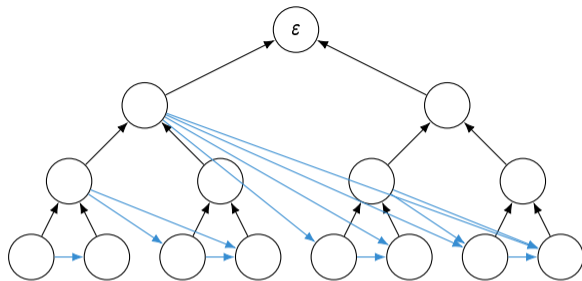
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Concurrent/Subsequent Work.

Chung et al. [CFHL21]: also gave a comparable security bounds for the PoSW in the pqROM

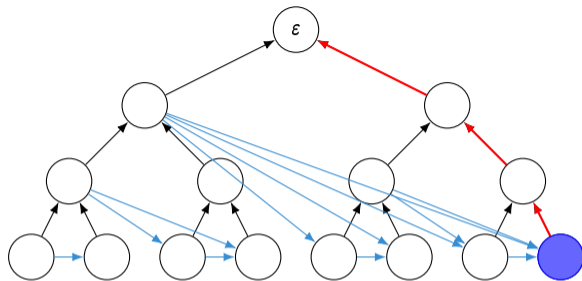
The [CP18] Construction



statement being proved,
e.g., final exam solution

For all leaf nodes v , add an edge (v, w) for any w that is a left sibling of a node on the path from v to the root

The [CP18] Construction

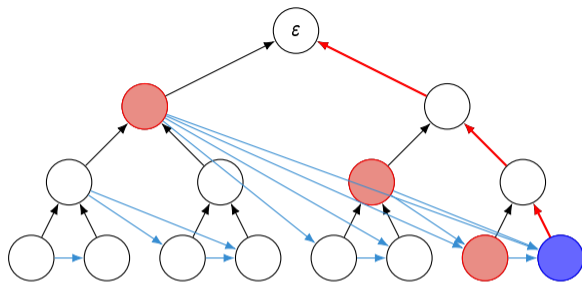


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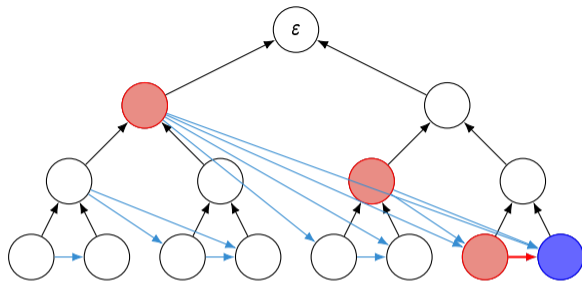


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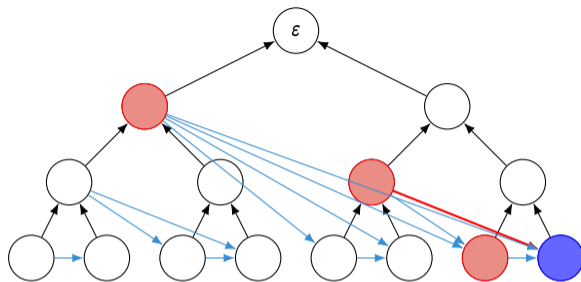


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For all leaf nodes ℓ , add an edge $\ell \rightarrow p(\ell)$ if $p(\ell)$ is a left sibling of a node on the path from ℓ to the root

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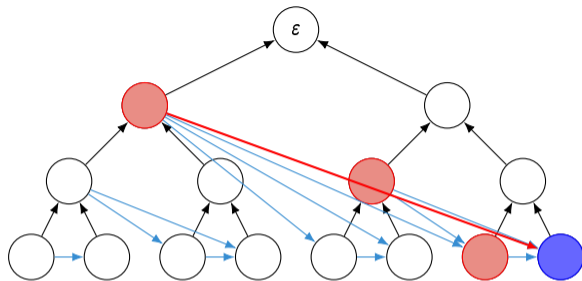


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For all leaf nodes v , add an edge $(v, p(v))$ to the root

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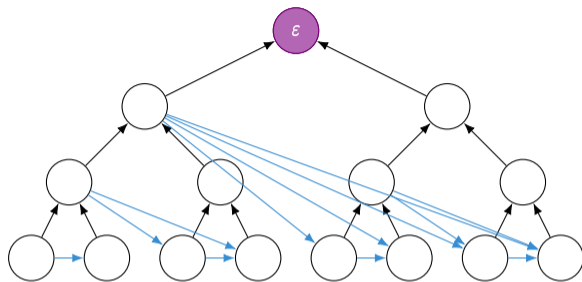


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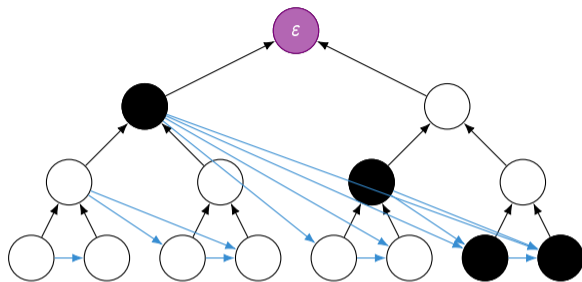
Each node has a label, a hash of its parents

The label of root node forms a commitment of all the other nodes

Verifier can audit the prover by forcing the prover to open certain labels

Show that they are locally consistent

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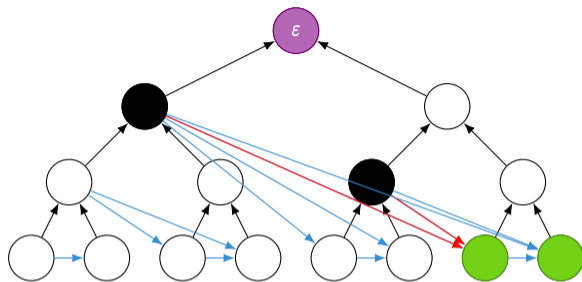
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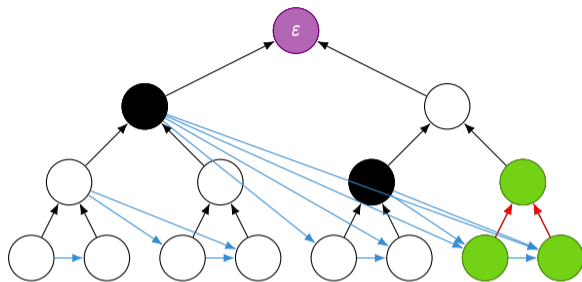
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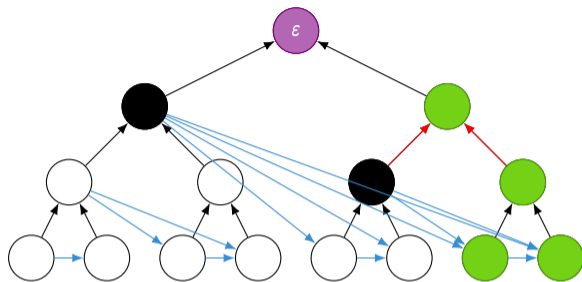
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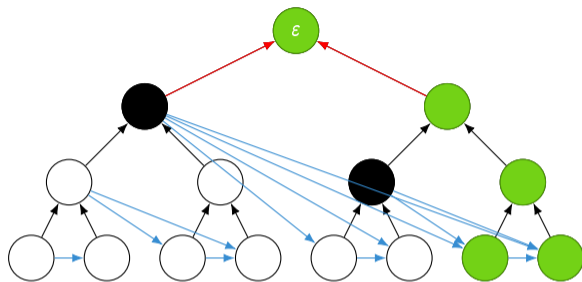
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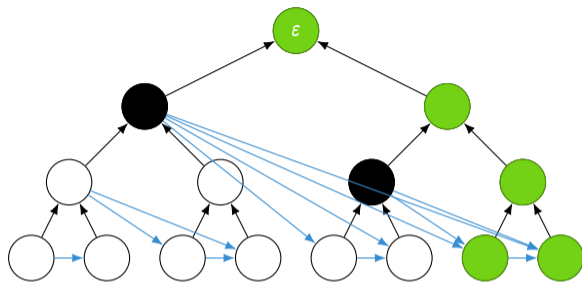
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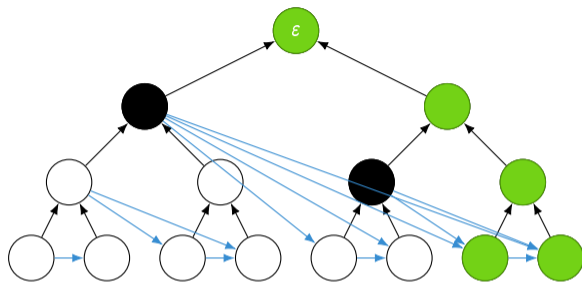
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Audit process: interactive or non-interactive (Fiat-Shamir)

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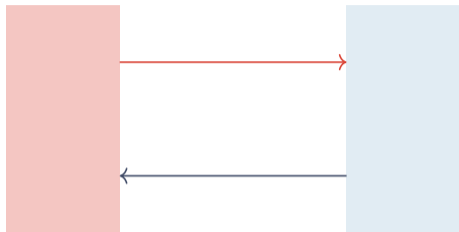
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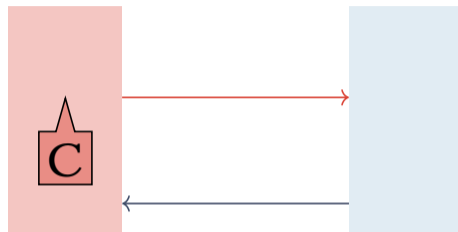
Any classical ROM attacker that produces a valid PoSW in time T must produce a long T -sequence

ROM vs qROM [BDF 11]

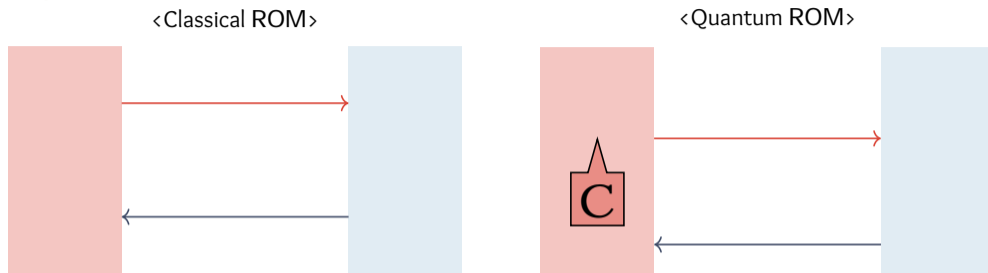
<Classical ROM>



<Quantum ROM>



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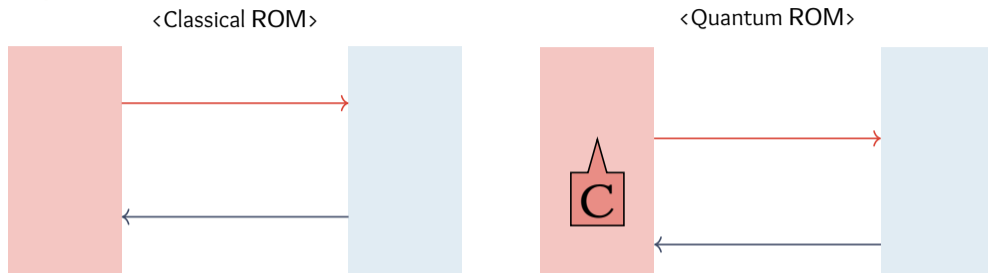


Security proofs are much more challenging in the qROM

Programmability & Extractability (ROM: 4, qROM: 8)

Recording quantum queries?

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Compressed Oracle Technique [Zha19]: **change of view** (compressed phase oracle (CPhsO))

Compressed Phase Oracle (CPhsO)

A database \mathcal{D} , where $f(x) = \mathcal{D}(x)$.

How to view a random oracle?

Classical: databases of known I/O pairs & unknown I/O pairs don't appear

Quantum: superposition over databases (known I/O pairs indeterminate)

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After q queries,

The state can be viewed as



where

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A database D , where D is a set of n pairs (x_i, y_i) .

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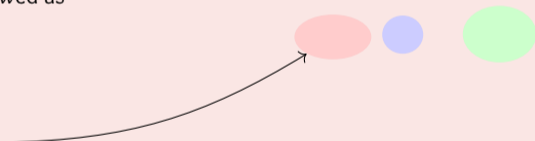
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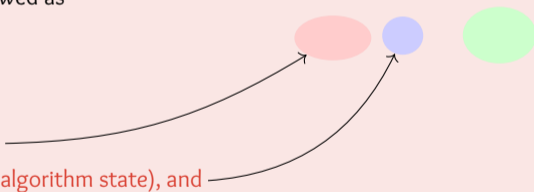
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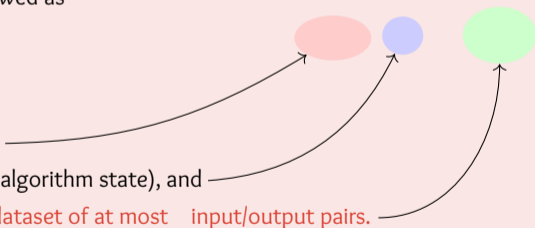
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a compressed dataset of at most q input/output pairs.



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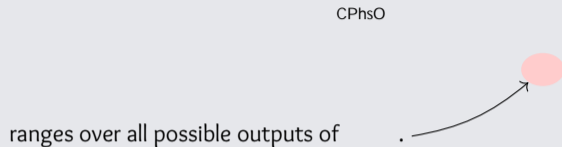
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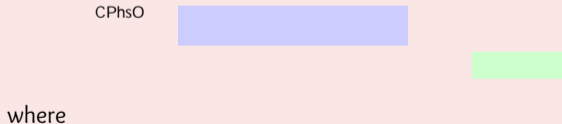


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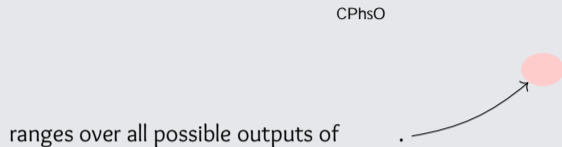


Example: Parallel Query (simplest case)

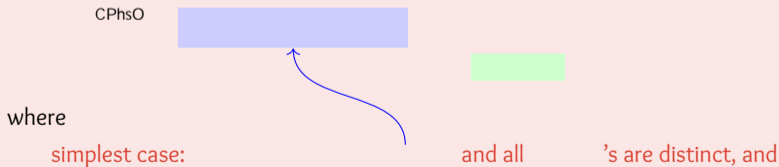


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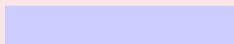
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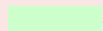
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's range over all possible outputs of



and all

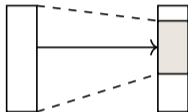
's are distinct, and

's for each .

Notations

Given a database
such that:

, define a directed graph on nodes



PATH

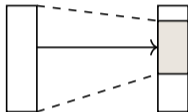
contains a path of length (set of databases), and

$\hat{\text{PATH}}$

PATH (set of basis states).

Notations

Given a database
such that:



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PATH

contains a path of length (set of databases), and

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PATH (set of basis states).

contains an
-sequence of length

PATH

Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

Lemma

$|\psi\rangle$: an initial state, and let C_{PhsO} . Then $\langle \psi | \hat{P}_{\text{PATH}} \rangle \geq \epsilon$ implies $\langle C_{\text{PhsO}} \psi | \hat{P}_{\text{PATH}} \rangle \geq \epsilon - \epsilon^2$.

Interpretation/Intuition:

$\langle \psi | \hat{P}_{\text{PATH}} \rangle$: 2-norm of the projection of $|\psi\rangle$ onto \hat{P}_{PATH} , i.e.,

$$\frac{\langle \psi | \hat{P}_{\text{PATH}} \rangle}{\|\hat{P}_{\text{PATH}}\|}$$

If we start with the state that is nearly orthogonal to \hat{P}_{PATH} , then after applying the oracle C_{PhsO} , the resulting state is also nearly orthogonal to \hat{P}_{PATH} .

Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

Lemma

ρ : an initial state, and let

CPhsO

. Then

\hat{P} PATH

\hat{P} PATH

ϵ .

Basic proof idea: split the states into **good** and **bad** part. (in this talk, suppose that \hat{P} and ρ are distinct for simplicity)

and all

Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

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CPhsO

Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

Lemma

ρ : an initial state, and let \mathcal{C} be a CPhsO. Then \exists a \mathcal{P} -PATH ρ and all ρ .

Basic proof idea: split the states into **good** and **bad** part. (in this talk, suppose that ρ and all ρ are distinct for simplicity)

CPhsO

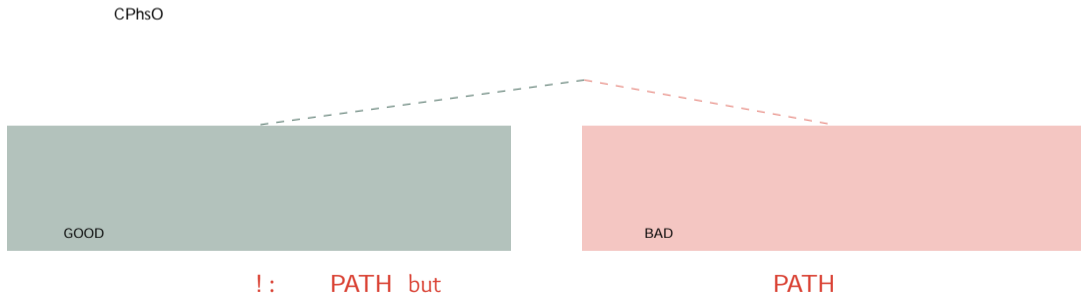


Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

Lemma

ρ : an initial state, and let \mathcal{C} PhsO . Then \exists PATH $\hat{\rho}$ PATH ϵ .

Basic proof idea: split the states into **good** and **bad** part. (in this talk, suppose that ρ and all ρ_i are distinct for simplicity)



Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

Lemma

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CPhsO

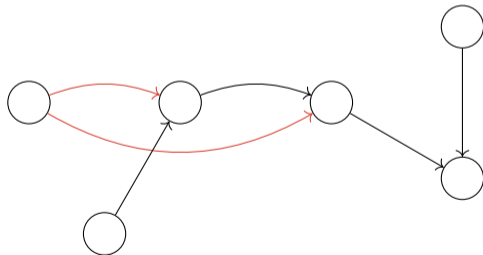
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PATH

PATH

_____.

Proof by example:



Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

Lemma

ρ : an initial state, and let

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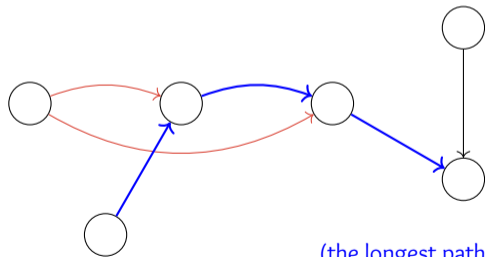
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PATH

PATH

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Proof by example:



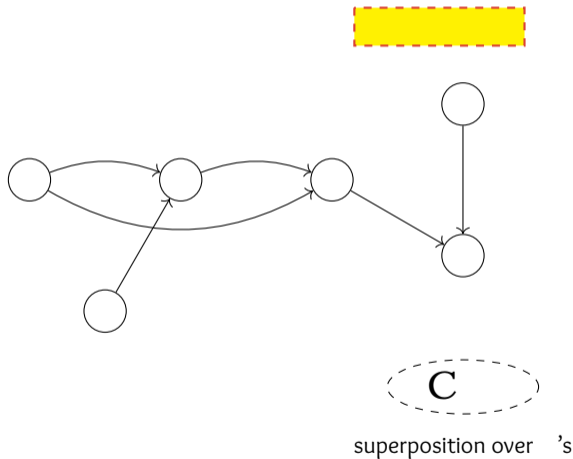
(the longest path)=

PATH but

PATH

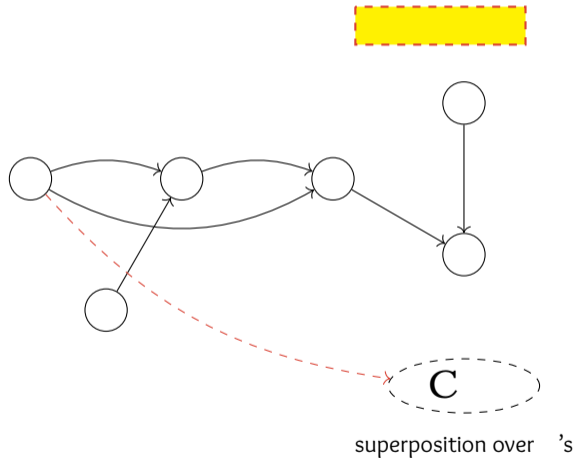
Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

Suppose we have **one query**: (where $\epsilon > 0$). Then the updated database is:



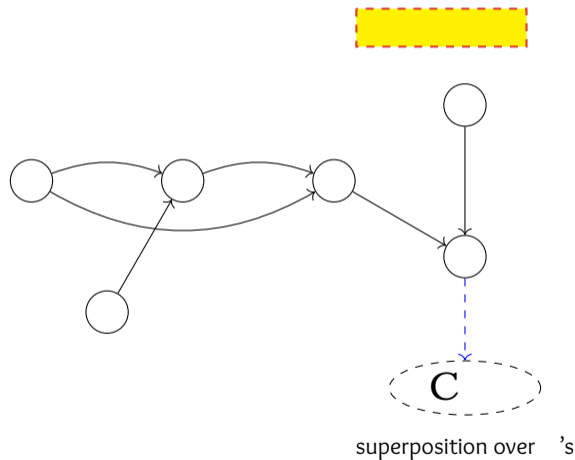
Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

Suppose we have **one query**: Q (where Q is a quantum circuit). Then the updated database is:



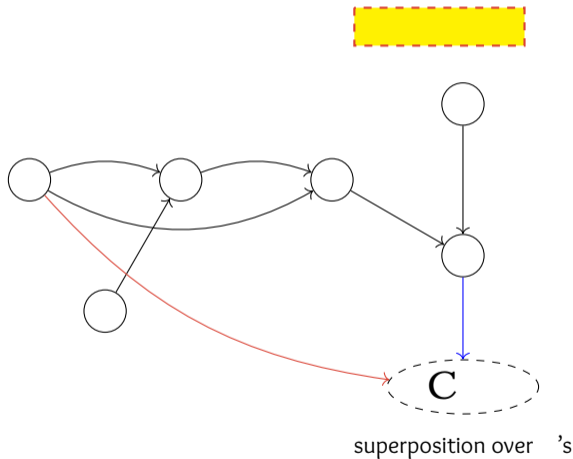
Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

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Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

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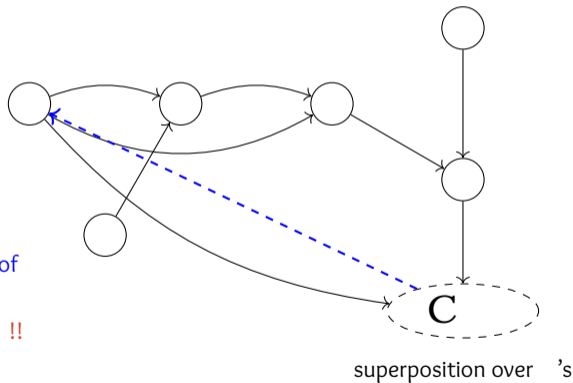


Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

Suppose we have **one query:** (where). Then the updated database is:



- ! if:
- back edges from to some (e.g.,)
- PATH but PATH !!

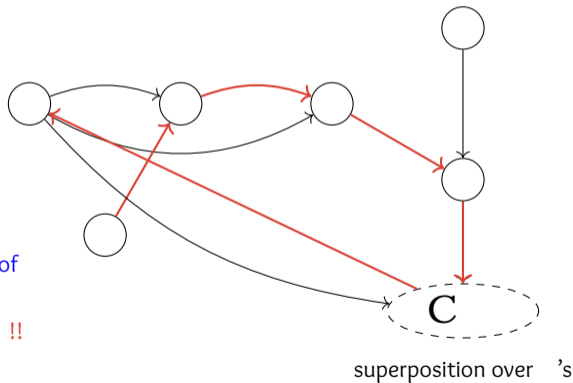


Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

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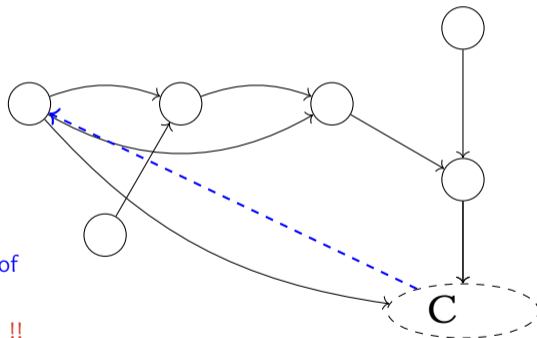


- ! if:
- back edges from to some (e.g.,)
- PATH but PATH !!



Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

Suppose we have **one query**: (x, y) (where $y = f(x)$). Then the updated database is:



- ! if:
- back edges from C to some x (e.g., $x = f^{-1}(C)$)
- PATH but **PATH !!**

superposition over C 's

Key observation:

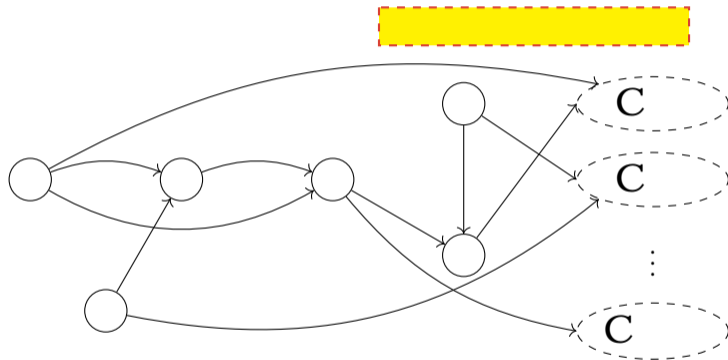
The fraction of such C 's is negligible! (ϵ out of 2^n)

Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

For a **parallel query**:

(where

and all x_i 's are distinct for simplicity),

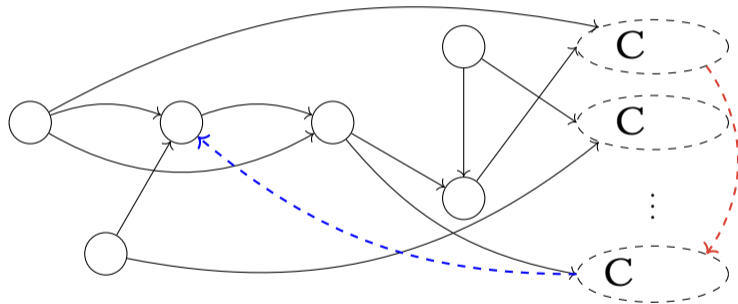


Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

For a **parallel query**:

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! if:

1. **internal edges** between x_i 's, and
2. **back edges** from x_j to some x_i .

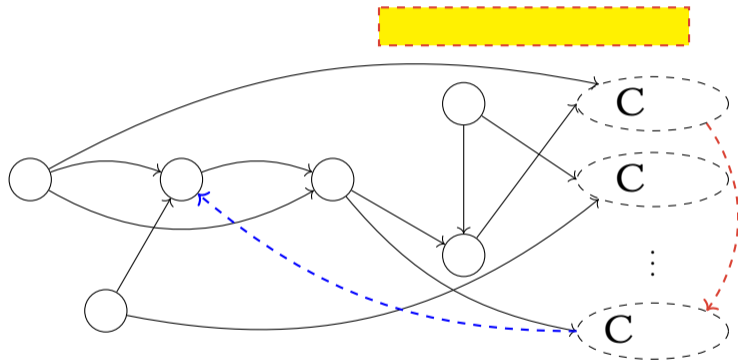
PATH but
PATH !!

Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

For a **parallel query**:

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! if:

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PATH but
PATH !!

Key observation: The fraction of such x_i 's is negligibly small! (ϵ out of $1/\epsilon$ for each)

Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

We have shown: parallel queries in a single round,

PATH

PATH



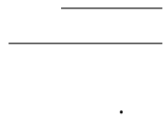
Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

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PATH

Throughout rounds: parallel queries in each round, with



Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

We have shown: parallel queries in a single round,

\hat{P} PATH

\hat{P} PATH

Throughout r rounds: parallel queries in each round, with

By triangle inequality,

\hat{P} PATH

measures a database in **PATH** with probability at most ϵ ,

i.e., can produce an **ϵ -sequence of length r** with only negligible probability ϵ .

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We only considered the simplest case in this talk - see the paper for the full security proof!

Proof Ideas: Hardness of Producing an ϵ -sequence in a Quantum Setting

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measures a database in **PATH** with probability at most ϵ ,

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We only considered the simplest case in this talk - see the paper for the full security proof!

Security of a non-interactive PoSW: similar argument using the result above - details in the paper

($M, f, K, \epsilon, \delta, \epsilon < \delta \leq \epsilon @ K; \delta C A \neq K$)

Concluding Remarks

Takeaways.

PoSW allows a prover to convince a resource-bounded verifier that the prover invested a substantial amount of sequential time to produce a valid proof.

Concluding Remarks

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PoSW allows a prover to convince a resource-bounded verifier that the prover invested a substantial amount of sequential time to produce a valid proof.

Any attacker in the pqROM making Q total queries in T sequential rounds cannot find an ℓ -sequence of length ℓ except with negligible probability ϵ .

Concluding Remarks

Takeaways.

PoSW allows a prover to convince a resource-bounded verifier that the prover invested a substantial amount of sequential time to produce a valid proof.

Any attacker in the pqROM making q total queries in T sequential rounds cannot find an α -sequence of length n except with negligible probability ϵ .

Any attacker in the pqROM making q total queries in sequential time cannot produce a valid non-interactive PoSW ([CP18] construction) except with negligible probability ϵ .

Concluding Remarks

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Open Questions.

Can we tighten the security bound from ϵ to ϵ^2 ?

Concluding Remarks

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PoS_W allows a prover to convince a resource-bounded verifier that the prover invested a substantial amount of sequential time to produce a valid proof.

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Establishing security for larger ℓ : can we extract more than ℓ challenges from a single RO output?

Concluding Remarks

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Can we prove for an interactive PoSW?

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Can we extend our security proof for other PoSW constructions?

Concluding Remarks

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PoSW allows a prover to convince a resource-bounded verifier that the prover invested a substantial amount of sequential time to produce a valid proof.

Any attacker in the pqROM making q total queries in T sequential rounds cannot find an ℓ -sequence of length ℓ except with negligible probability ϵ .

Any attacker in the pqROM making q total queries in sequential time cannot produce a valid non-interactive PoSW ([CP18] construction) except with negligible probability ϵ .

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Can we prove for an interactive PoSW?

Can we extend our security proof for other PoSW constructions?

Can techniques extend to other primitives, e.g., Proofs of Space, Memory-Hard Functions, etc.?

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