Adversarial Robustness of Streaming Algorithms through Importance Sampling

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Model

❖ Input: Elements of an underlying data set S, which arrives sequentially and *adversarially.* Adversary can choose future inputs after seeing previous outputs by honest algorithm
❖ Output: Evaluation (or approximation) of a given function
❖ Goal: Use space *sublinear* in the size m of the input S

❖ Surprising separation between "classic" streaming model where the stream input is fixed but the order of the updates may be given adversarially
❖ Hardt and Woodruff [HW13] showed that Linear sketches are NOT robust to adversarial attacks, must use $\Omega(n)$ space by giving an attack on AMS F_2 algorithm

Applications / Motivations

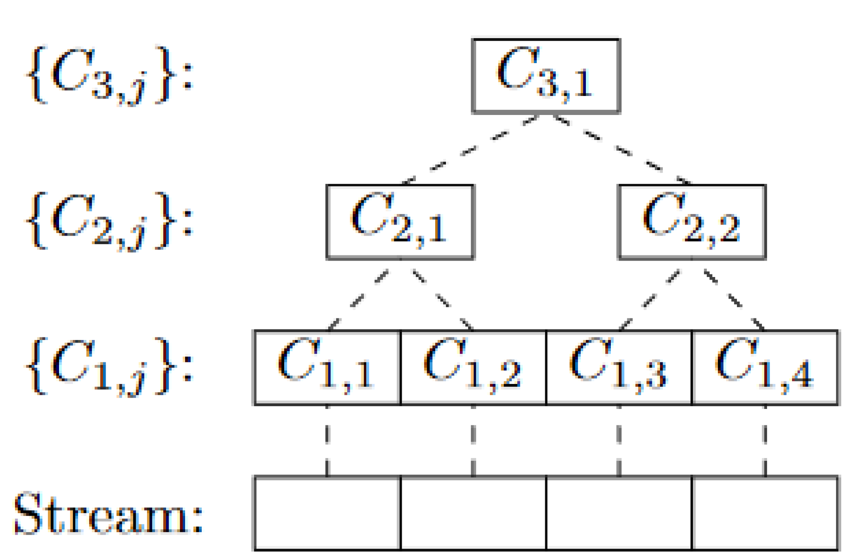
❖ Adversarial machine learning: ML problems where the input is chosen by an adversary
❖ Database queries: For multiple queries to a database, each query may depend on the responses to the previous queries
❖ Transparency of Algorithms: Internal state of honest algorithms may be entirely revealed or otherwise compromised

Coresets

❖ Coreset: Returns an ϵ-approximation on a query space

❖ Merge and reduce framework: Each $C_{1,j}$ is an $\frac{\epsilon}{\log n}$ coreset of the corresponding partition of the substream

❖ Applications: k-means clustering, k-median clustering, projective clustering, principal component analysis, Bayesian logistic regression, generative adversarial networks, k-line center, M-estimators



Row Sampling Algorithms for Linear Algebra

❖ Row-arrival model: $M_1, \ldots, M_n \in R^d$ rows of a matrix M
❖ Sample each row based on its "importance" to obtain $(1+\epsilon)$-approximate solutions to each problem

❖ Linear Regression: Output $x \in R^d$ to minimize $\|Mx - b\|_2$
❖ Spectral Sparsification / Subspace Embedding: Output $A \in R^{m \times d}$ so that $\|Mx\|_2 \approx \|Ax\|_2$ for all $x \in R^d$ and $m \ll n$
❖ Low-Rank Approximation: Output $A \in R^{m \times d}$ so that $\|M - MP\|_F \approx \|A - AP\|_F$ for all rank k projection matrices P
❖ L_1 Subspace Embedding: Output $A \in R^{m \times d}$ so that $\|Mx\|_1 \approx \|Ax\|_1$ for all $x \in R^d$ and $m \ll n$

Edge Sampling Algorithms for Graphs

❖ Edge-arrival model: $e_1, \ldots, e_m$ edges of a graph G
❖ Sample each edge based on its "importance" to obtain $(1+\epsilon)$-approximate solutions to each problem

❖ Graph Sparsification: Output H so that $Cut_H(S, V \setminus S) \approx Cut_G(S, V \setminus S)$ for any $S \subset V$

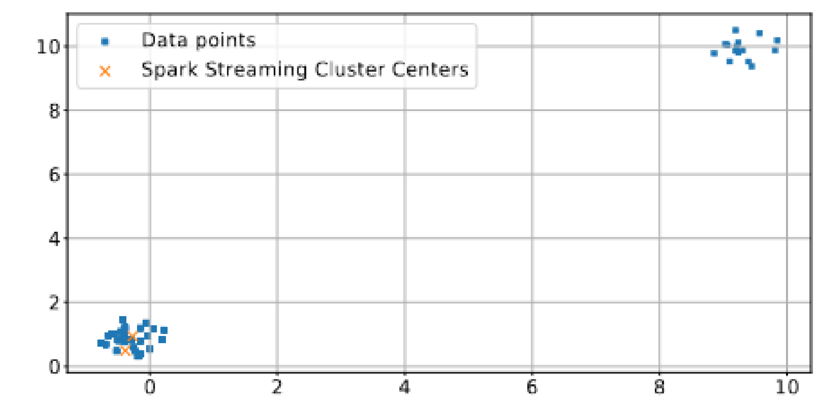
Results and Related Work

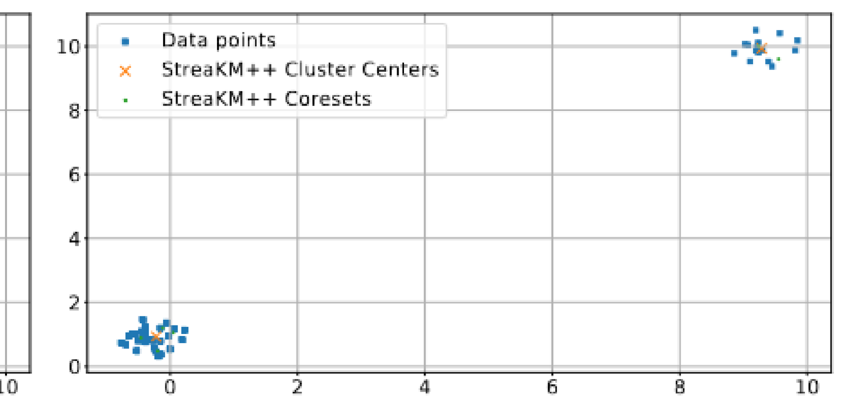
❖ Our result: Importance sampling based algorithms are adversarially robust!
❖ Intuition: Importance is a robust metric and sampling based algorithms use public randomness that is independent of previous randomness

❖ Corollary: Merge-and-reduce is adversarially robust
❖ Corollary: Row sampling algorithms are adversarially robust
❖ Corollary: Edge sampling algorithm is adversarially robust

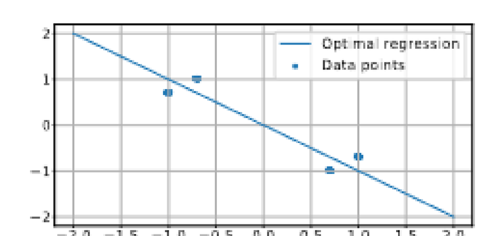
Empirical Evaluations

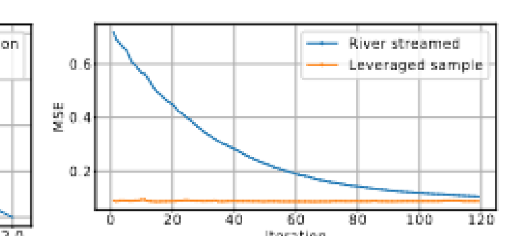
❖ Streaming k-means clustering: a series of point batches where all points except the last batch are randomly sampled from a two-dimensional standard normal distribution. Points in the last batch sampled but around a distant center

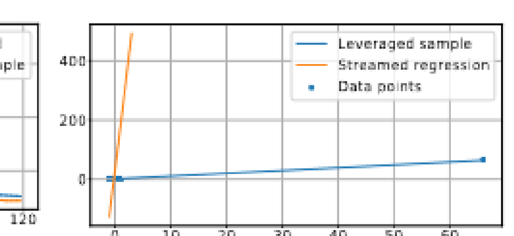


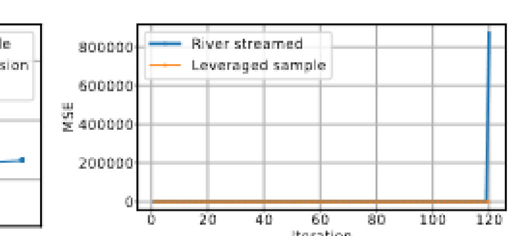


❖ Streaming linear regression: all batches except the last one are sampled around a constellation of four points in the plane such that the optimal regression line is of -1 slope through the origin. The last batch is at (L, L), far from the origin so the resulting optimal regression line has slope 1 through the origin

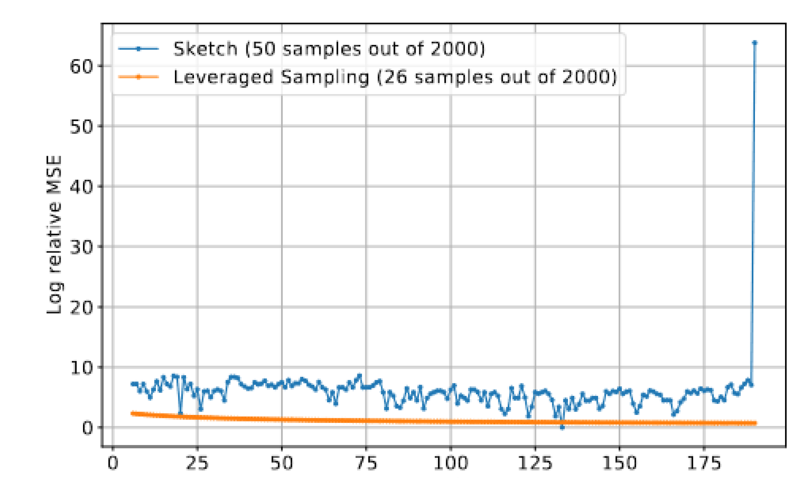








❖ Sampling vs. sketching: For a random unit sketching matrix S (each of its elements is sampled from $\{-1,1\}$ with equal probability), we create an adversarial data stream M such that its columns are in the nullspace of S for linear regression



References

❖ [HW13] Moritz Hardt, David P. Woodruff: How robust are linear sketches to adaptive inputs? STOC 2013: 121-130
❖ [KMNS21] Haim Kaplan, Yishay Mansour, Kobbi Nissim, Uri Stemmer: Separating Adaptive Streaming from Oblivious Streaming. CRYPTO 2021 (to appear)